Table 1 Variation of over-all specular component of reflectance for perfectly reflecting surface 1[a]

H/L	$\overline{(\rho^s/\rho)}_1$						$I_{2\lambda}$
	$\sigma = 0.75\mu$			$\sigma = 1.5\mu$			a, b
	280°K	590°K	760°K	280°K	590°K	760°K	
	0.785		0.419	0.497		0.150	1, 19
$\frac{1}{6}$	0.774	0.500	0.394	0.474	0.202	0.131	1, 0
	0.769		0.383	0.465		0.122	20, −19
	0.764		0.361	0.446		0.101	1, 19
$\frac{1}{2}$	0.747	0.433	0.332	0.419	0.148	0.087	1, 0
	0.737		0.315	0.403		0.079	20, −19

[a] In this table, $W = L$; the intensity of radiation leaving surface 2 is diffuse and varies linearly with x, $I_{2\lambda}(x/L, y/L) = a + b(x/L)$. The spectral distribution corresponds to blackbody emission at the indicated temperature.

energy leaving each surface and

$$g_{ij\lambda}\pi A_i F_{ij} = \int_{A_i}\int_{A_j} g_i(\theta_1', \sigma/\lambda) K_{ij}\, dA_j\, dA_i \tag{7}$$

Note that in writing Eq. (7) it was assumed that $I_{j\lambda}$ is uniform over each surface. If this assumption is not valid, the surface can be subdivided into a number of smaller zones. For a highly reflecting enclosure, the weighting factors w_j may be justifiably assumed to be equal because of a large number of interreflections. In other cases, only those w_j need be retained for which surfaces A_j make the most contribution to $\overline{(\rho^s/\rho)}_i$.

In the other limiting situation when the enclosure consists of highly emitting surfaces, Eq. (5) can be expressed as[7]

$$\overline{(\rho^s/\rho)}_i = \int_0^\infty \sum_{j=1}^n E_{bj\lambda}(\varepsilon_{ji\lambda} g_{ij\lambda} \rho_{ij\lambda}) F_{ij}\, d\lambda \Bigg/ \int_0^\infty \sum_{j=1}^n E_{bj\lambda}(\varepsilon_{ji\lambda}\rho_{ij\lambda}) F_{ij}\, d\lambda \tag{8}$$

where the term in brackets represents the geometric thermal radiation characteristics[6] defined analogously to Eq. (7). In practical problems, it is convenient to evaluate these characteristics at some mean value of the angles. As a guide, if the distance between surfaces is more than five times the largest dimension of the source surface the error in applying the inverse square law is less than 1% (Ref. 8). For the special case when $\rho_\lambda(\theta') = \text{const}$ evaluation of ε and g at some mean angle θ_{ji} (the angle formed by normal to A_j and line joining A_j with A_i) can be expressed as

$$\overline{(\rho^s/\rho)}_i = \sum_{j=1}^n F_{ij} \int_0^\infty \varepsilon_{j\lambda}(\theta_{ji}) g_i(\theta_{ij}', \sigma/\lambda) E_{bj\lambda}\, d\lambda \Bigg/ \sum_{j=1}^n F_{ij} \int_0^\infty \varepsilon_{j\lambda}(\theta_{ji}) E_{bj\lambda}\, d\lambda \tag{9}$$

Example

The quantitative effects of various parameters on $\overline{(\rho^s/\rho)}_i$ are illustrated in Fig. 2 and Table 1. The configuration considered is shown as an insert in Fig. 2 and consists of two square surfaces. For highly reflecting material, the results were essentially the same. The trends were similar for other directional distributions of $I_{2\lambda}$. The results are given only for $y/L = \frac{7}{16}$ since for $y/L = \frac{1}{16}$ the difference was small.

Comparison of the results (including those not presented here) show that the local as well as the over-all component of reflectance is not sensitive to the variation in intensity leaving surface 2. The local specular component of reflectance shows a large variation across the surface for a relatively close configuration ($H/L = \frac{1}{6}$), especially for the large roughness and higher temperature. In spite of the large local variations, the over-all values (Table 1) are very close to the local values at locations where most of the energy is incident in near normal directions. This is due to the fact that the intensity incident at oblique angles has only a small contribution to the over-all specular component of reflectance. Use of the specular component of reflectance calculated in the manner presented in this Note yielded reasonably good agreement between predictions and data.[7]

References

[1] Sparrow, E. M. and Cess, R. D., *Radiation Heat Transfer*, Brooks/Cole Publishing, Belmont, Calif., 1966.

[2] Schornhorst, J. R. and Viskanta, R., "Effect of Direction and Wavelength Dependent Surface Properties on Radiant Heat Transfer," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, pp. 1450–1455.

[3] Edwards, D. K. and Bertak, I. V., "Imperfect Reflections in Thermal Radiation Transfer," AIAA Paper 70-860, Los Angeles, Calif., 1970.

[4] Beckmann, P. and Spizzichino, A., *The Scattering of Electromagnetic Waves from Rough Surfaces*, Macmillan, New York, 1963.

[5] Houchens, A. F. and Hering, R. G., "Bidirectional Reflectance of Rough Metal Surfaces," *Thermophysics of Spacecraft and Planetary Bodies*, edited by G. B. Heller, Academic Press, New York, 1967, pp. 65–90.

[6] Bevans, J. T. and Edwards, D. K., "Radiation Exchange in an Enclosure with Directional Wall Properties," *Transactions of the ASME, Ser. C: Journal of Heat Transfer*, Vol. 87, No. 3, Aug. 1965, pp. 388–396.

[7] Toor, J. S., "An Experimental and Analytical Study of Spectral and Directional Effects on Radiant Heat Transfer," Ph.D. thesis, 1971, Purdue Univ., Ind.

[8] Moon, P., *The Scientific Basis of Illuminating Engineering*, Dover, New York, 1961.

Trajectory Determination from Shock Arrival Times

WALTER P. REID*
Naval Ordnance Laboratory, Silver Spring, Md.

AN object traveling in air at a speed greater than that of sound generates a shock wave. Under ideal conditions, if the velocity is constant, the wave has the shape of a right circular cone except in the neighborhood of the object. In this Note a method will be given for calculating the position of the object at any instant and its vector velocity from measurements of the times at which the shock wave arrives at some microphones of known locations. Solutions to this problem have been

Received September 27, 1971.

* Research Mathematician.

presented previously.[1,2] For some applications, the method of Ref. 2 might be preferred because with it the velocity vector of the moving object can be found without knowing the coordinates of the listening stations. However, at least three such stations are needed. The present Note gives an alternate method of computation in which only two observation posts are required. This can be advantageous in some cases.

Let t be the time at which the shock wave arrives at a particular point. Finite difference approximations for $\partial t/\partial x$, $\partial t/\partial y$ and $\partial t/\partial z$ can be obtained by having two or more microphones along the x, y and z axes through the point, respectively. This gives ∇t, the gradient of t. But the shock wave travels with the speed of sound, C, in the direction perpendicular to the wave front. Hence ∇t is in the direction of the vector $\mathbf{C}$, and its magnitude is $1/C$. That is,

$$C^2 \nabla t = \mathbf{C} \tag{1}$$

$$(\partial t/\partial x)^2 + (\partial t/\partial y)^2 + (\partial t/\partial z)^2 = 1/C^2 \tag{2}$$

If the speed of sound is known, then only two of the partial derivatives of t need be measured. The third one can be calculated, except for sign, by means of Eq. (2). Thus a cluster of microphones can serve as an observation post to determine ∇t, and therefore $\mathbf{C}$.

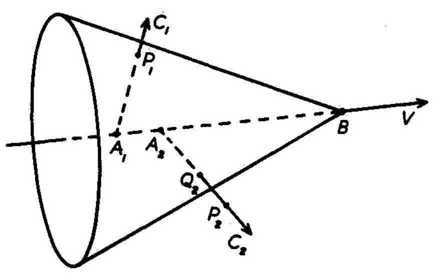

Fig. 1 The object at B, moving with velocity V greater than C, creates a shock cone as shown.

Refer now to Fig. 1. It shows a portion of the shock cone generated by an object moving along path $A_1 A_2 B$ with velocity $\mathbf{V}$. Observers are at P_1 and P_2. The shock wave has just arrived at P_1 and Q_2. The time will be designated as t_1, and the arrival time at P_2 will be called t_2. Hence

$$\mathbf{Q}_2 = \mathbf{P}_2 - \mathbf{C}_2(t_2 - t_1) \tag{3}$$

This gives the point Q_2. Let τ_1 be the time required for the shock wave to travel from A_1 to P_1, and τ_2 the time it spends going from A_2 to Q_2. Then

$$\mathbf{V}(\tau_1 - \tau_2) = \mathbf{C}_1 \tau_1 + \mathbf{Q}_2 - \mathbf{P}_1 - \mathbf{C}_2 \tau_2 \tag{4}$$

But the cosine of the angle between $\mathbf{V}$ and $\mathbf{C}$ is equal to C/V. Therefore $\mathbf{V} \cdot \mathbf{C} = C^2$. So dotting Eq. (4) with $\mathbf{C}_2$ and then $\mathbf{C}_1$ gives

$$(C^2 - \mathbf{C}_1 \cdot \mathbf{C}_2)\tau_1 = \mathbf{C}_2 \cdot (\mathbf{Q}_2 - \mathbf{P}_1) \tag{5}$$

$$(C^2 - \mathbf{C}_1 \cdot \mathbf{C}_2)\tau_2 = \mathbf{C}_1 \cdot (\mathbf{P}_1 - \mathbf{Q}_2) \tag{6}$$

These equations give τ_1 and τ_2 unless $\mathbf{C}_1 = \mathbf{C}_2$. If $(\mathbf{C}_1 + \mathbf{C}_2) \cdot (\mathbf{Q}_2 - \mathbf{P}_1)$ is not zero, then τ_1 and τ_2 will be different, and $\mathbf{V}$ can be obtained from Eq. (4). Also,

$$\mathbf{A}_1 = \mathbf{P}_1 - \mathbf{C}_1 \tau_1 \tag{7}$$

Thus the point A_1 on the path of the moving object, and the time $t_1 - \tau_1$ when it was there, are also known.

References

[1] Zaroodny, S. J., "Trajectory Indicator—A Proposal," *SIAM Journal on Applied Mathematics*, Vol. 14, No. 6, 1966, pp. 1366–1389.

[2] Reid, W. P., "Acoustic Tracking of Supersonic Objects," *AIAA Journal*, Vol. 8, No. 5, May 1970, pp. 973–974.

Prediction of Turbulent Boundary-Layer Separation Influenced by Blowing

ARNOLD POLAK*

Naval Ordnance Laboratory, Silver Spring, Md.

Introduction

RESULTS of a study of turbulent boundary-layer separation were presented in Ref. 1. In an effort to obtain additional data on flow separation at high speeds, another test series was carried out in the NOL Hypersonic Tunnel to study the effect of mass addition. The attempt to predict separation of a turbulent boundary layer over a cone-flare configuration with injection by an extension of the method of Ref. 1 resulted in this Note.

Consider the supersonic, axisymmetric turbulent boundary layer over a circular cone which is fed by gas injected normally through a porous section of the wall and subsequently its separation is induced by a conical flare. Separation is assumed to commence over the solid portion of the wall past the porous section. The analysis is concerned primarily with prediction of the turbulent boundary-layer characteristics ahead of separation. When these are known, the separation point is determined by the same procedure as used in Ref. 1. The analysis rests on some concepts introduced in Ref. 2 at low speeds, modified to be applicable to the high-speed flow. It is assumed that: a) the injected gas, having the same composition as the gas in the boundary layer is perfect; b) the blowing rate does not reach values at which the assumptions of the boundary-layer theory are violated.

Turbulent Boundary-Layer Characteristics

For constant flow conditions along the outer edge of the boundary layer the momentum integral equation is

$$c_f/2 = d(x\delta_2)/x\,dx - \rho_w v_w/\rho_e u_e \tag{1}$$

For a turbulent boundary layer with no blowing the skin-friction coefficient, c_f, and the Reynolds number based on momentum thickness, Re_2, are related by

$$c_{fo} Re_{2o}{}^{1/4} = C_1 \tag{2}$$

C_1 is, for given freestream conditions and wall temperature, a constant evaluated by the reference-temperature method. The subscript zero refers to an impermeable wall.

Consider first flow past a cone with permeable wall over its whole length. The boundary layer is assumed to be fully turbulent. From Eqs. (1) and (2) we get

$$2d(Re\,Re_2)/C_1(Re\,Re_2)^{-1/4} = Re^{5/4}(b+\psi)\,dRe \tag{3}$$

where Re denotes the Reynolds number based on distance x, and ψ and the blowing parameter b are defined by

$$\psi \equiv (c_f/c_{fo})_{Re_2}, \quad b \equiv 2\rho_w v_w/\rho_e u_e c_{fo} \tag{4}$$

In definition (4) both c_f and c_{fo} are evaluated at the same freestream conditions and wall temperature but at corresponding stations such that, Re_2, is the same. For constant b and ψ integration of Eq. (3) yields

$$(Re\,Re_2)^{5/4} = \tfrac{5}{18} C_1 Re^{9/4}(b+\psi) \tag{5}$$

Evaluate Eq. (5) at the same value of Re with $b = 0$ and divide this into Eq. (5) to get

$$Re_2 = (b+\psi)^{4/5} Re_{2o} \tag{6}$$

The skin-friction coefficients for no blowing evaluated from Eq. (2)

Received September 27, 1971; revision received November 2, 1971. The author wishes to acknowledge the contribution of D. L. Merritt, Naval Ordnance Laboratory, Applied Aerodynamics Division, who furnished the experimental data used in this Note.

Index categories: Boundary-Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow.

* Research Aerospace Engineer, Applied Aerodynamics Division; also Associate Professor, University of Cincinnati, Department of Aerospace Engineering. Member AIAA.